Antenna Theory and Design

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Chapter 5
Microstrip antenna

Chapter 5 Microstrip antenna

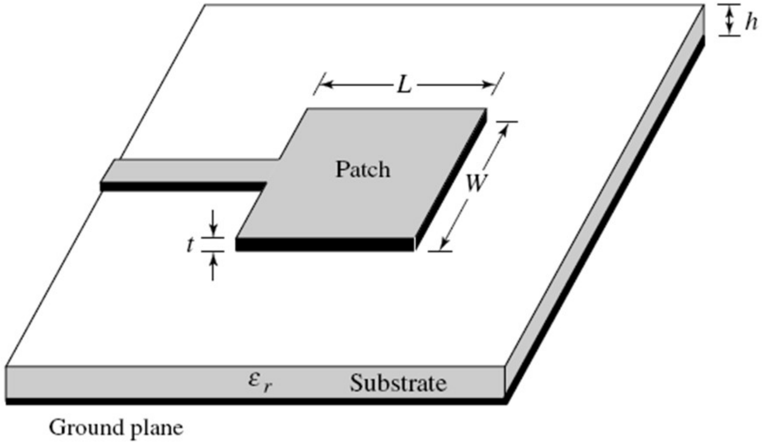
- Introduction of microstrip antenna (MSA)
- Feeding techniques of MSA
- Methods of analysis
- Design of a rectangular patch

1. Introduction

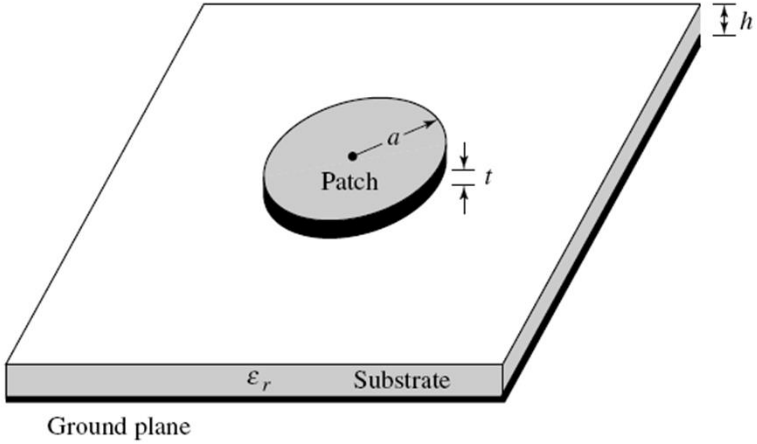
- In high-performance aircraft, spacecraft, satellite, and missile applications, where size, weight, cost, performance, ease of installation, and aerodynamic profile are constraints, low-profile antennas may be required. Presently there are many other government and commercial applications, such as mobile radio and wireless communications, that have similar specifications.
  - To meet these requirements, microstrip antennas can be used.

- Major operational disadvantages of microstrip antennas are their low efficiency, low power, high Q (sometimes in excess of 100), poor polarization purity, poor scan performance, spurious feed radiation and very narrow frequency bandwidth, which is typically only a fraction of a percent or at most a few percent.

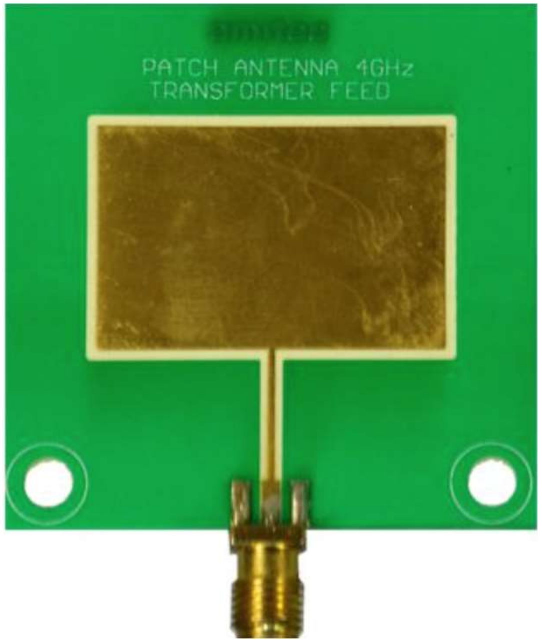
- Deschamps first proposed the concept of the MSA in 1953. However, practical antennas were developed by Munson and Howell in the 1970s.

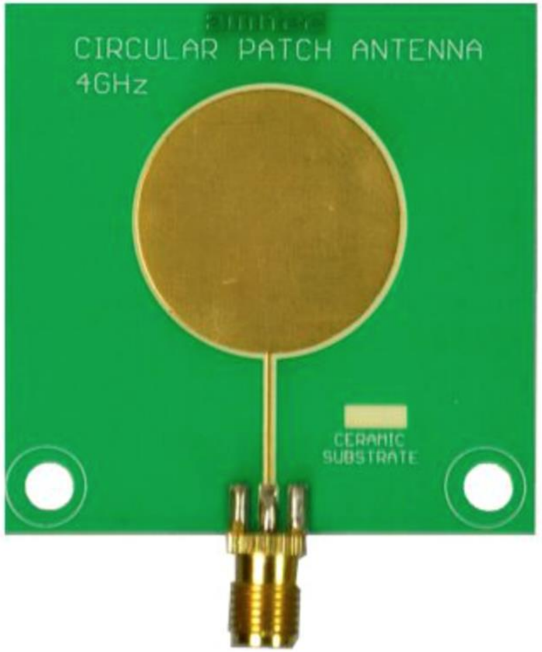


(a) Rectangular



(b) Circular

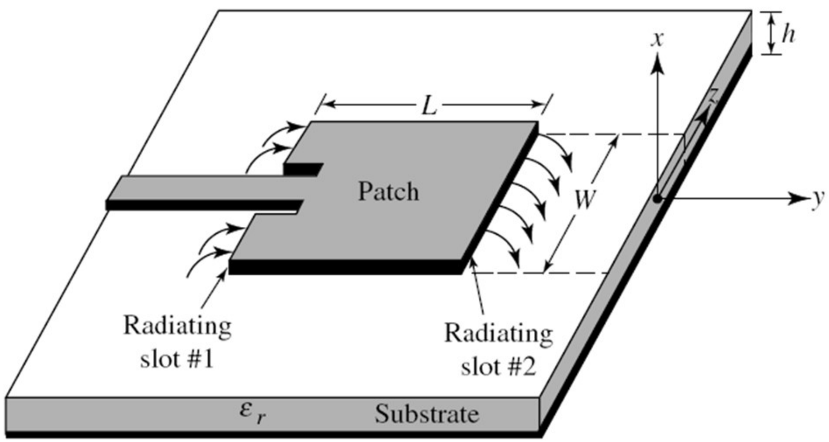




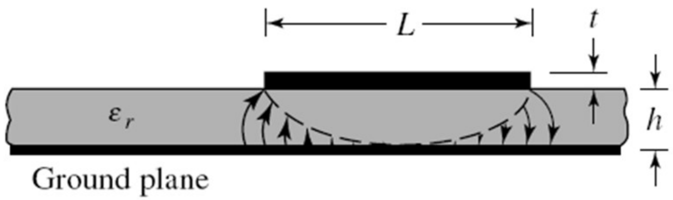
1.1 Basic characteristics

- An MSA in its simplest form consists of a radiating patch on one side of a dielectric substrate and a ground plane on the other side.

- Normally, it consists of a very thin ($h << \lambda_0$,where λ_0 is the free-space wavelength) metallic strip (patch) placed a small fraction of a wavelength ($h << \lambda_0$, usually $0.003\lambda_0 \leq h \leq 0.05\lambda_0$) above a ground plane. The microstrip patch is designed so its pattern maximum is normal to the patch (broadside radiator).



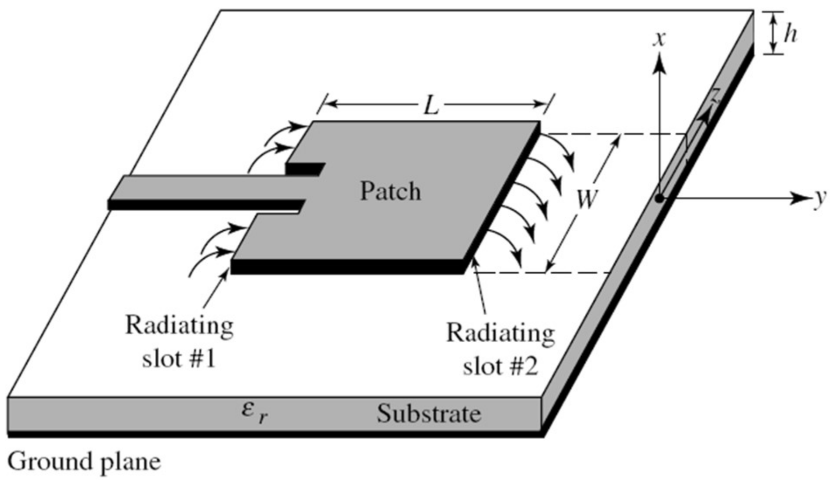
(a) Microstrip antenna



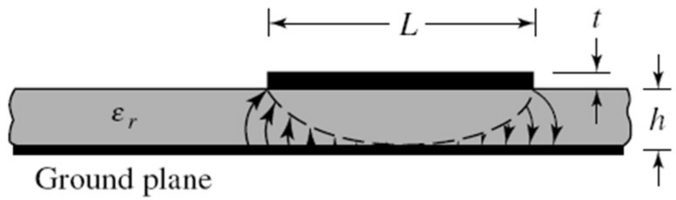
(b) Side view

1.1 Basic characteristics

- Radiation from the MSA can occur from the fringing fields between the periphery of the patch and the ground plane. The length L of the rectangular patch for the fundamental TM_{10} mode excitation is slightly smaller than $\lambda/2$, where λ is the wavelength in the dielectric medium. Which in terms of free-space wavelength λ_0 is given as $\lambda_0/\sqrt{\varepsilon_e}$, where $\lambda_0/\sqrt{\varepsilon_e}$ is the effective dielectric constant of a microstrip line of width W.
- The value of ε_e is slightly less than the dielectric constant ε_r of the substrate because the fringing fields from the patch to the ground plane are not confined in the dielectric only, but are also spread in the air.



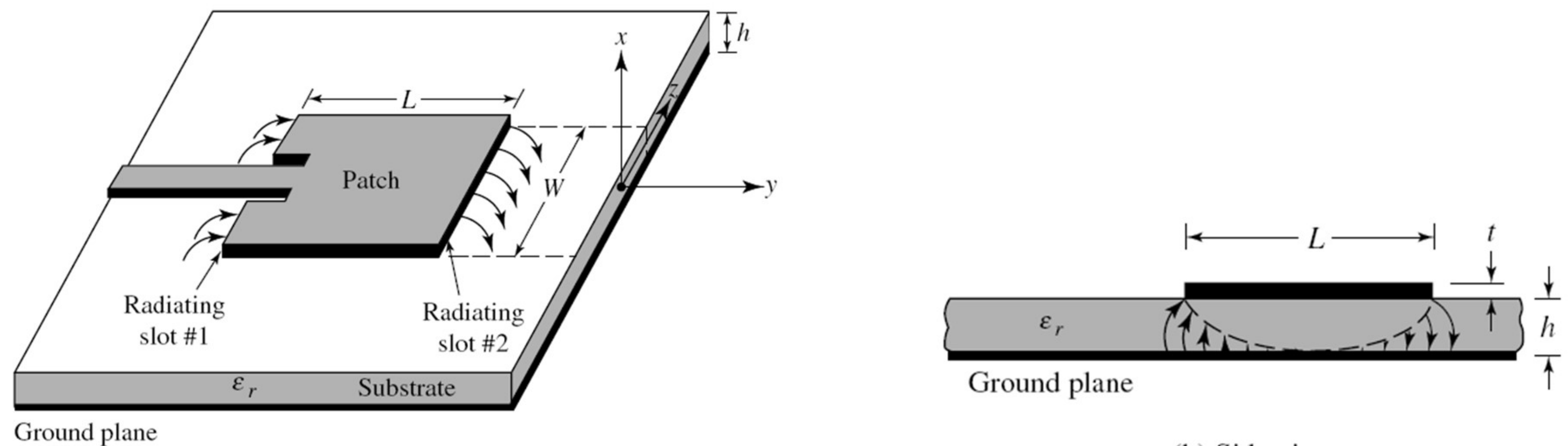
(a) Microstrip antenna



(b) Side view

1.1 Basic characteristics

- For a rectangular patch, the length L of the element is usually $\lambda/3 < L < \lambda/2$.
- There are numerous substrates that can be used for the design of microstrip antennas, and their dielectric constants are usually in the range of $2.2 \leq \varepsilon_r \leq 12$.
- The ones that are most desirable for good antenna performance are thick substrates whose dielectric constant is in the lower end of the range because they provide better efficiency, larger bandwidth, loosely bound fields for radiation into space, but at the expense of larger element size.
- Thin substrates with higher dielectric constants are desirable for microwave circuitry because they require tightly bound fields to minimize undesired radiation and coupling, and lead to smaller element sizes; however, because of their greater losses, they are less efficient and have relatively smaller bandwidths.

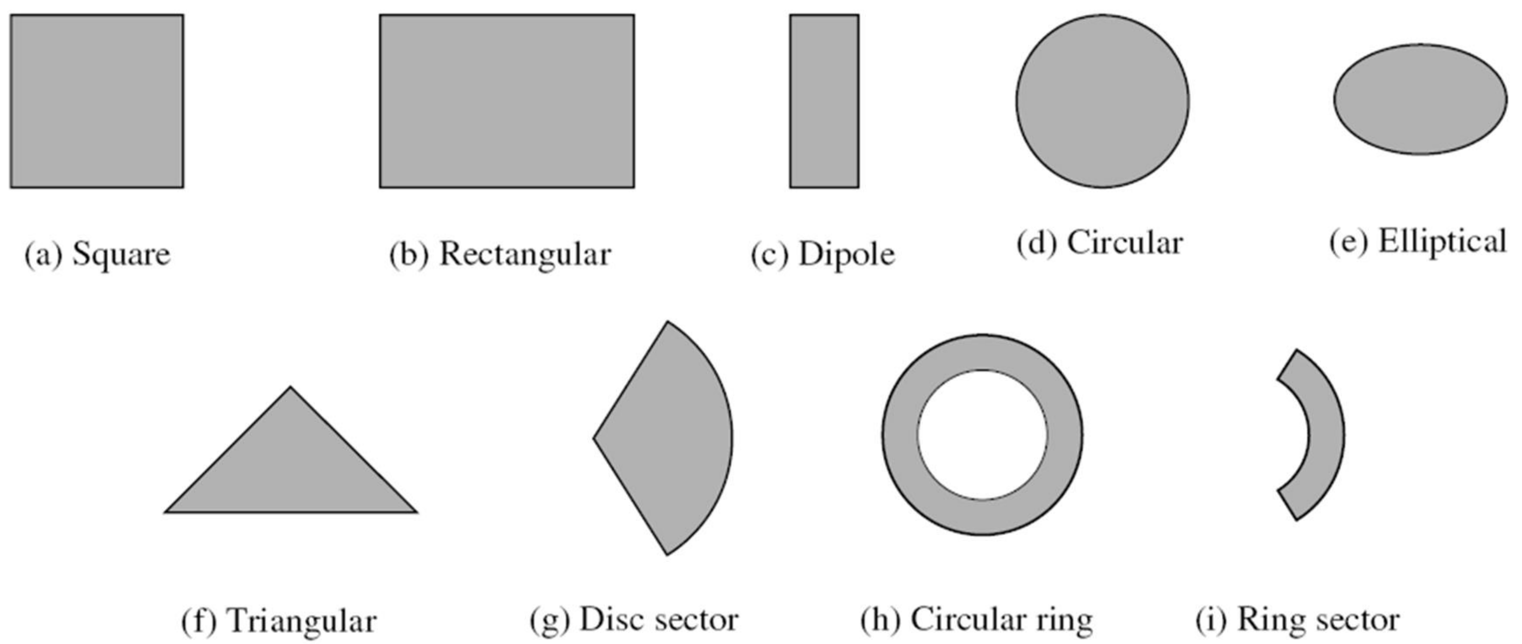


(a) Microstrip antenna

(b) Side view

1.1 Basic characteristics

- Often microstrip antennas are also referred to as *patch antennas*. The radiating elements and the feed lines are usually photoetched on the dielectric substrate.
- The radiating patch may be square, rectangular, thin strip (dipole), circular, elliptical, triangular, or any other configuration.

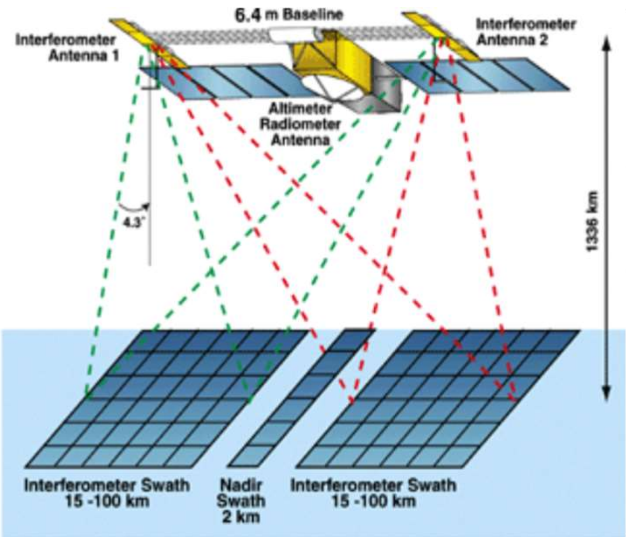


1.2 Advantages and disadvantages

- Advantages:
  - ✓ They are lightweight and have a small volume and a low-profile planar configuration.
  - ✓ They can be made conformal to the host surface.
  - ✓ Their ease of mass production using printed-circuit technology leads to a low fabrication cost.
  - ✓ They are easier to integrate with other MICs on the same substrate.
  - ✓ They allow both linear polarization and CP.
  - ✓ They can be made compact for use in personal mobile communication.
  - ✓ They allow for dual- and triple-frequency operations.
- Disadvantages:
  - ✓ Narrow BW;
  - ✓ Lower gain;
  - ✓ Low power-handling capability.
- MSAs have narrow BW, typically 1–5%, which is the major limiting factor for the widespread application of these antennas. Increasing the BW of MSAs has been the major thrust of research in this field, and broad BW up to 70% has been achieved.

1.3 Applications

- The telemetry and communications antennas on missiles need to be thin and conformal and are often MSAs.
- Radar altimeters use small arrays of microstrip radiators.
- Other aircraft-related applications include antennas for telephone and satellite communications.
- Microstrip arrays have been used for satellite imaging systems.
- Patch antennas have been used on communication links between ships or buoys and satellites.
- Smart weapon systems use MSAs because of their thin profile.
- Pagers, the global system for mobile communication (GSM), and the global positioning system (GPS) are major users of MSAs.



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Typical Applications of MSAs

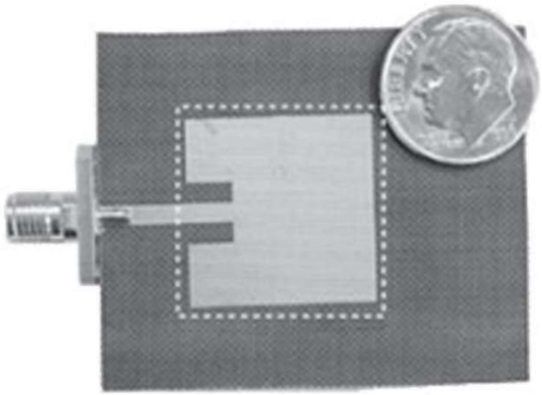
System	**Application**
Aircraft and ship antennas	Communication and navigation, altimeters, blind landing systems
Missiles	Radar, proximity fuses, and telemetry
Satellite communications	Domestic direct broadcast TV, vehicle-based antennas, communication
Mobile radio	Pagers and hand telephones, man pack systems, mobile vehicle
Remote sensing	Large lightweight apertures
Biomedical	Applicators in microwave hyperthermia
Others	Intruder alarms, personal communication, and so forth

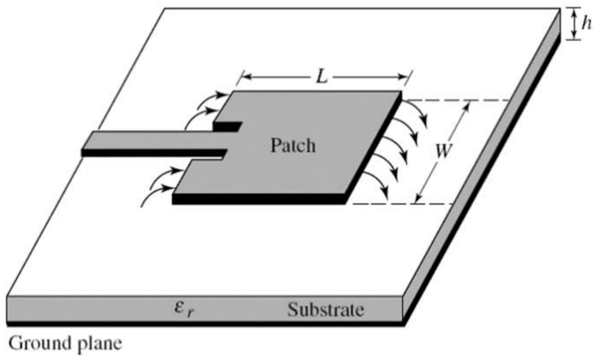
2. Feeding techniques

- Feeding technique influences the input impedance and characteristics of the antenna, and is an important design parameter.
- The MSA can be *excited directly* either by a coaxial probe or by a microstrip line. It can also be *excited indirectly* using electromagnetic coupling or aperture coupling and a coplanar waveguide feed, in which case there is no direct metallic contact between the feed line and the patch.
- The four most popular are：
    - the microstrip line
    - coaxial probe
    - aperture coupling
    - proximity coupling.

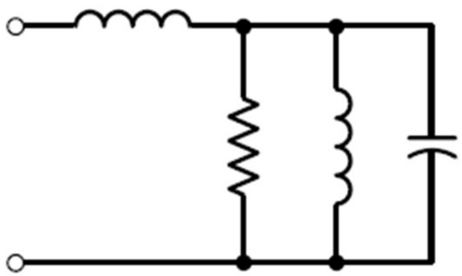
2.1 Microstrip line feed

- The microstrip feed line is also a conducting strip, usually of much smaller width compared to the patch.
- Advantages: the microstrip-line feed is easy to fabricate, simple to match by controlling the inset position and rather simple to model. This feed arrangement has the advantage that it can be etched on the same substrate, so the total structure remains planar.
- Disadvantages: But as the substrate thickness increases, surface waves and spurious feed radiation increase, which for practical designs limit the bandwidth (typically 2–5%). The drawback is the radiation from the feed line, which leads to an increase in the cross-polar level. Also, in the millimeter-wave range, the size of the feed line is comparable to the patch size, leading to increased undesired radiation.





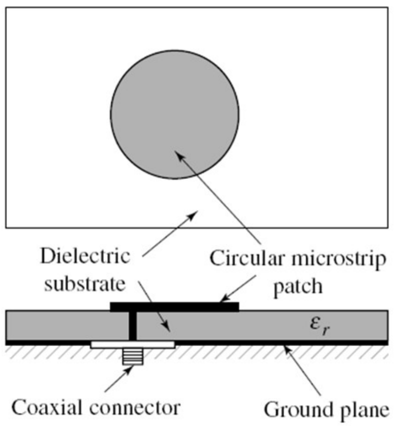
(a) Microstrip line feed



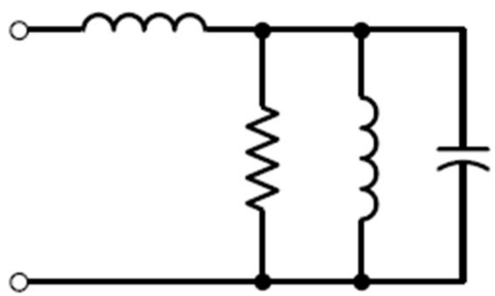
(a) Microstrip line

2.2 Probe feed

- Coaxial-line feeds, where the inner conductor of the coax is attached to the radiation patch while the outer conductor is connected to the ground plane, are also widely used.
- Advantages: The coaxial probe feed is easy to fabricate and match, and it has low spurious radiation. The main advantage of this feed is that it can be placed at any desired location inside the patch to match with its input impedance.
- Disadvantages: However, it also has narrow bandwidth and it is more difficult to model, especially for thick substrates $(h > 0.02\lambda_0)$. The disadvantages are that the hole has to be drilled in the substrate and that the connector protrudes outside the bottom ground plane, so that it is not completely planar. Also, this feeding arrangement makes the configuration asymmetrical.



(b) Probe feed

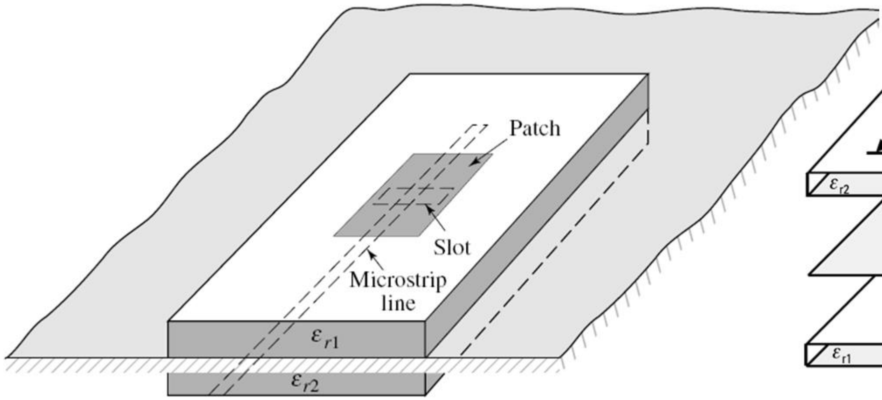

(b) Probe

2.3 Aperture coupling

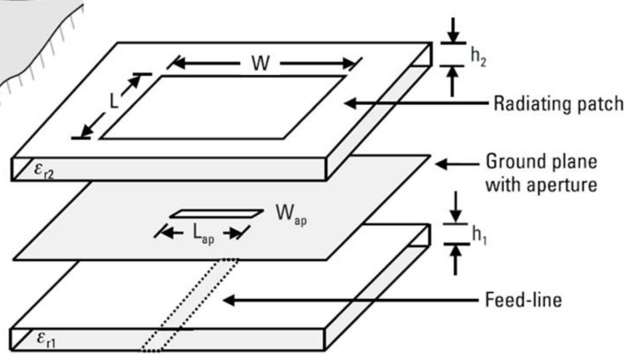
- Problems of direct feeding: Both the microstrip feed line and the probe possess inherent asymmetries which generate higher order modes which produce cross-polarized radiation. For thick substrates, which are generally employed to achieve broad BW, both the above methods of direct feeding the MSA have problems. In the case of a coaxial feed, increased probe length makes the input impedance more inductive, leading to the matching problem. For the microstrip feed, an increase in the substrate thickness increases its width, which in turn increases the undesired feed radiation.
- The indirect feed solves these problems.
- The aperture coupling is the most difficult of all four to fabricate and it also has narrow bandwidth. However, it is somewhat easier to model and has moderate spurious radiation.
- In the aperture-coupled MSA configuration, the field is coupled from the microstrip line feed to the radiating patch through an electrically small aperture or slot cut in the ground plane.

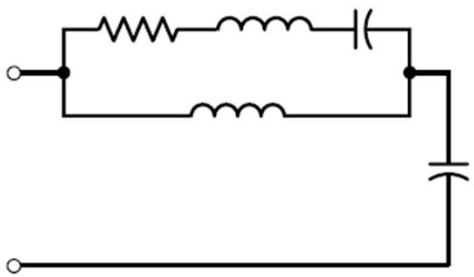
2.3 Aperture coupling

- The aperture coupling consists of two substrates separated by a ground plane.
- On the bottom side of the lower substrate there is a microstrip feed line whose energy is coupled to the patch through a slot on the ground plane separating the two substrates. This arrangement allows independent optimization of the feed mechanism and the radiating element.
- Typically a high dielectric material is used for the bottom substrate, and thick low dielectric constant material for the top substrate. The ground plane between the substrates also isolates the feed from the radiating element and minimizes interference of spurious radiation for pattern formation and polarization purity.



(c) Aperture-coupled feed




(c) Aperture-coupled

2.3 Aperture coupling

- For this design, the substrate electrical parameters, feed line width, and slot size and position can be used to optimize the design. Typically matching is performed by controlling the width of the feed line and the length of the slot.

- The slot aperture can be either resonant or nonresonant. The resonant slot provides another resonance in addition to the patch resonance thereby increasing the BW at the expense of an increase in back radiation. As a result, a nonresonant aperture is normally used.

- The coupling through the slot can be modeled using the theory of Bethe, which is also used to account for coupling through a small aperture in a conducting plane.

- In this theory the slot is represented by an equivalent normal electric dipole to account for the normal component (to the slot) of the electric field, and an equivalent horizontal magnetic dipole to account for the tangential component (to the slot) mag- netic field.

2.3 Aperture coupling

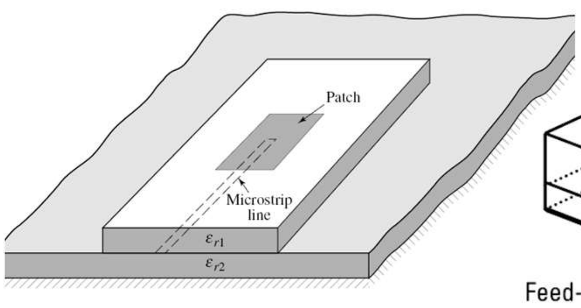
- If the slot is centered below the patch, where ideally for the dominant mode the electric field is zero while the magnetic field is maximum, the magnetic coupling will dominate. Doing this also leads to good polarization purity and no cross-polarized radiation in the principal planes.

- The performance is relatively insensitive to small errors in the alignment of the different layers.

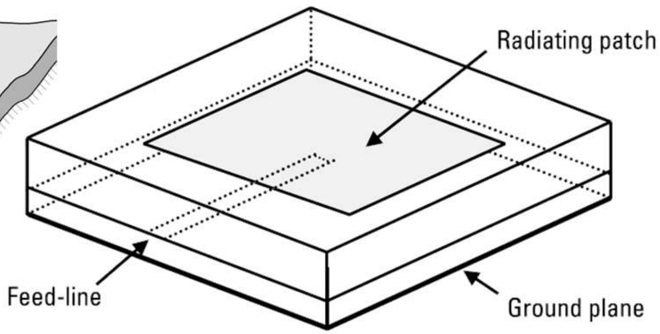
- The substrate parameters of the two layers can be chosen separately for optimum antenna performance.

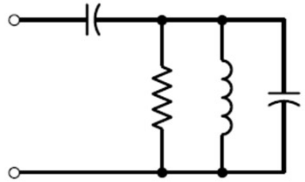
2.4 Proximity coupling

- The electromagnetic coupling is also known as proximity coupling.
- The feed line is placed between the patch and the ground plane, which is separated by two dielectric media. The length of the feeding stub and the width-to-line ratio of the patch can be used to control the match.
- The advantages of this feed configuration include the elimination of spurious feed-network radiation; the choice between two different dielectric media, one for the patch and the other for the feed line to optimize the individual performances; and an increase in the BW due to the increase in the overall substrate thickness of the MSA. The proximity coupling has the largest bandwidth (as high as 13 percent), is somewhat easy to model and has low spurious radiation.
- The disadvantages are that the two layers need to be aligned properly and that the overall thickness of the antenna increases.



(d) Proximity-coupled feed




(d) Proximity-coupled

3. Methods of analysis

- The MSA generally has a two-dimensional radiating patch on a thin dielectric substrate and therefore may be categorized as a two-dimensional planar component for analysis purposes. The analysis methods for MSAs can be broadly divided into two groups.
- In the first group, the methods are based on equivalent magnetic current distribution around the patch edges (similar to slot antennas). There are three popular analytical techniques:
  - The transmission line model;
  - The cavity model;
  - The multi-port network method (MNM).
- In the second group, the methods are based on the electric current distribution on the patch conductor and the ground plane (similar to dipole antennas, used in conjunction with full-wave simulation/numerical analysis methods). Some of the numerical methods for analyzing MSAs are listed as follows:
  - The method of moments (MoM);
  - The finite-element method (FEM);
  - The spectral domain technique (SDT);
  - The finite-difference time-domain (FDTD) method.

3.1 Transmission line model

- The transmission line model is very simple and helpful in understanding the basic performance of an MSA.
- The microstrip radiator element is viewed as a transmission line resonator with no transverse field variations (the field only varies along the length), and the radiation occurs mainly from the fringing fields at the open circuited ends.
- The patch is represented by two slots that are spaced by the length of the resonator.
- This model was originally developed for rectangular patches but has been extended for generalized patch shapes.
- Although the transmission line model is easy to use, all types of configurations can not be analyzed using this model since it does not take care of variation of field in the orthogonal direction to the direction of propagation.

3.2 Cavity model

- In the cavity model, the region between the patch and the ground plane is treated as a cavity that is surrounded by magnetic walls around the periphery and by electric walls from the top and bottom sides.
- Since thin substrates are used, the field inside the cavity is uniform along the thickness of the substrate. The fields underneath the patch for regular shapes such as rectangular, circular, triangular, and sectoral shapes can be expressed as a summation of the various resonant modes of the two-dimensional resonator.
- The fringing fields around the periphery are taken care of by extending the patch boundary outward so that the effective dimensions are larger than the physical dimensions of the patch.
- The effect of the radiation from the antenna and the conductor loss are accounted for by adding these losses to the loss tangent of the dielectric substrate.
- The far field and radiated power are computed from the equivalent magnetic current around the periphery.

3.3 MNM

- The MNM for analyzing the MSA is an extension of the cavity model.
- In this method, the electromagnetic fields underneath the patch and outside the patch are modeled separately.
- The patch is analyzed as a two-dimensional planar network, with a multiple number of ports located around the periphery.
- The multiport impedance matrix of the patch is obtained from its two-dimensional Green's function.
- The fringing fields along the periphery and the radiated fields are incorporated by adding an equivalent edge admittance network.
- The segmentation method is then used to find the overall impedance matrix.
- The radiated fields are obtained from the voltage distribution around the periphery.
- The above three analytical methods offer both simplicity and physical insight. These methods are accurate for regular patch geometries, but—except for MNM with contour integration techniques—they are not suited for arbitrary shaped patch configurations. For complex geometries, the numerical techniques described below are employed.

3.4 MoM

- The surface currents are used to model the microstrip patch, and volume polarization currents in the dielectric slab are used to model the fields in the dielectric slab.
- An integral equation is formulated for the unknown currents on the microstrip patches and the feed lines and their images in the ground plane.
- The integral equations are transformed into algebraic equations that can be easily solved using a computer.
- This method takes into account the fringing fields outside the physical boundary of the two-dimensional patch, thus providing a more exact solution.

3.5 FEM

- The FEM, unlike the MoM, is suitable for volumetric configurations. In this method, the region of interest is divided into any number of finite surfaces or volume elements depending upon the planar or volumetric structures to be analyzed.
- These discretized units, generally referred to as finite elements, can be any well-defined geometrical shapes such as triangular elements for planar configurations and tetrahedral and prismatic elements for three-dimensional configurations, which are suitable even for curved geometry. It involves the integration of certain basis functions over the entire conducting patch, which is divided into a number of subsections.
- The problem of solving wave equations with inhomogeneous boundary conditions is tackled by decomposing it into two boundary value problems, one with Laplace's equation with an inhomogeneous boundary and the other corresponding to an inhomogeneous wave equation with a homogeneous boundary condition.

3.6 FDTD method

- The FDTD method is well-suited for MSAs, as it can conveniently model numerous structural inhomogenities encountered in these configurations. It can also predict the response of the MSA over the wide BW with a single simulation.
- In this technique, spatial as well as time grid for the electric and magnetic fields are generated over which the solution is required.
- The spatial discretizations along three Cartesian coordinates are taken to be same.
- The E cell edges are aligned with the boundary of the configuration and H-fields are assumed to be located at the center of each E cell.
- Each cell contains information about material characteristics. The cells containing the sources are excited with a suitable excitation function, which propagates along the structure.
- The discretized time variations of the fields are determined at desired locations.
- Using a line integral of the electric field, the voltage across the two locations can be obtained. The current is computed by a loop integral of the magnetic field surrounding the conductor, where the Fourier transform yields a frequency response.

4. Design of a rectangular patch

- The rectangular patch is by far the most widely used configuration. It is very easy to analyze using both the transmission-line and cavity models, which are most accurate for thin substrates.

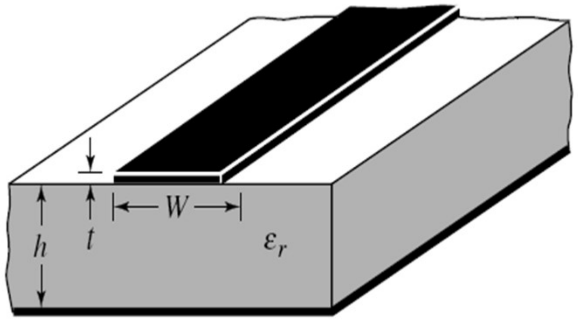
4.1 Transmission line model

- It was indicated earlier that the transmission-line model is the easiest of all but it yields the least accurate results and it lacks the versatility. However, it does shed some physical insight. Basically the transmission-line model represents the microstrip antenna by two slots, separated by a low-impedance Zc transmission line of length L.

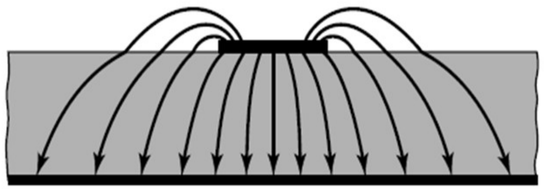
A. Fringing effects

- Because the dimensions of the patch are finite along the length and width, the fields at the edges of the patch undergo fringing.
- The amount of fringing is a function of the dimensions of the patch and the height of the substrate.
- For the principal E-plane (xy-plane) fringing is a function of the ratio of the length of the patch L to the height h of the substrate (L/h) and the dielectric constant ε_r of the substrate.

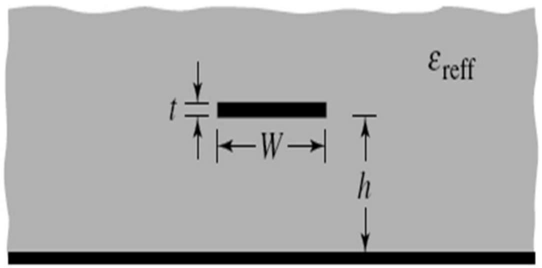
- Since for microstrip antennas L/h >>1, fringing is reduced; however, it must be taken into account because it influences the resonant frequency of the antenna. The same applies for the width.
- For a microstrip line shown in Figure (a), typical electric field lines are shown in Figure (b). This is a nonhomogeneous line of two dielectrics; typically the substrate and air.
- As can be seen, most of the electric field lines reside in the substrate and parts of some lines exist in air.
- As W/h >> 1 and ε_r >> 1, the electric field lines concentrate mostly in the substrate. Fringing in this case makes the microstrip line look wider electrically compared to its physical dimensions.
- Since some of the waves travel in the substrate and some in air, an effective dielectric constant ε_{reff} is introduced to account for fringing and the wave propagation in the line.



(a) Microstrip line

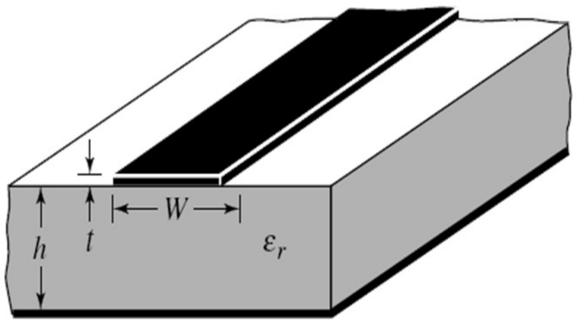


(b) Electric field lines

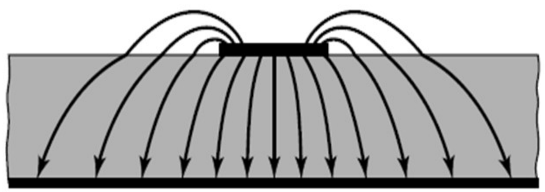


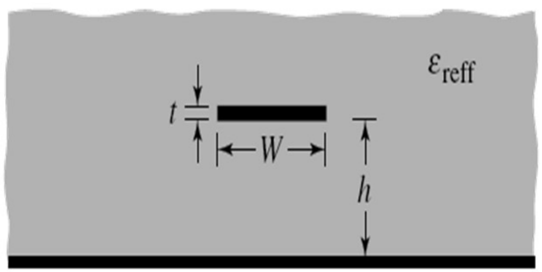
(c) Effective dielectric constant

- To introduce the effective dielectric constant, let us assume that the center conductor of the microstrip line with its original dimensions and height above the ground plane is embedded into one dielectric, as shown in Figure (c). The effective dielectric constant is defined as the dielectric constant of the uniform dielectric material so that the line of Figure (c) has identical electrical characteristics, particularly propagation constant, as the actual line of Figure (a).
- For a line with air above the substrate, the effective dielectric constant has values in the range of $1 < \varepsilon_{reff} < \varepsilon_r$.
- The effective dielectric constant is also a function of frequency.
- As the frequency of operation increases, most of the electric field lines concentrate in the substrate. Therefore the microstrip line behaves more like a homogeneous line of one dielectric (only the substrate), and the effective dielectric constant approaches the value of the dielectric constant of the substrate.

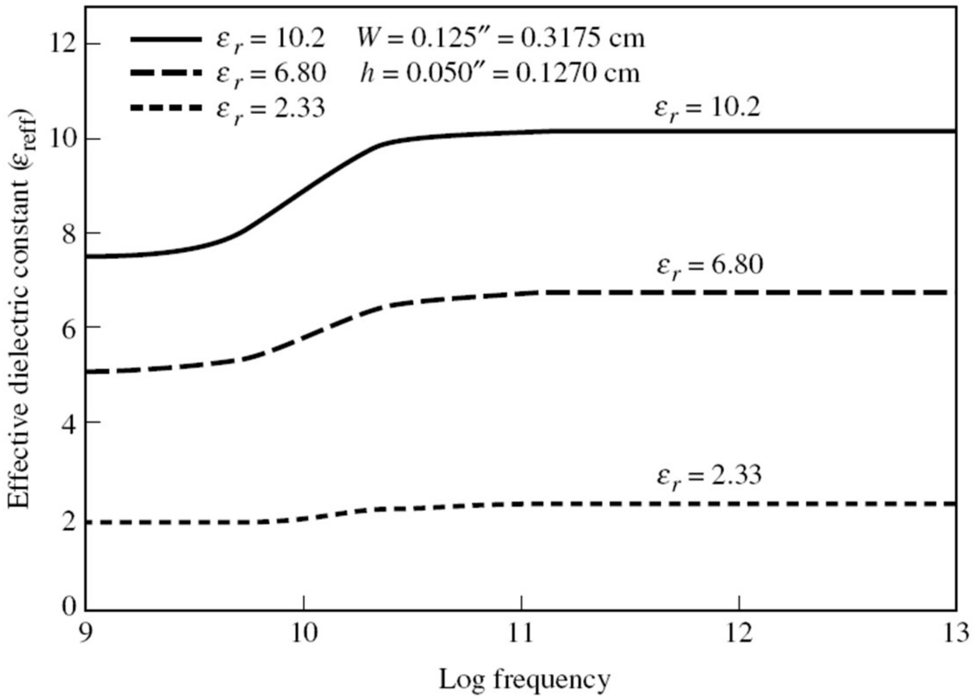


(a) Microstrip line


(b) Electric field lines



(c) Effective dielectric constant



For low frequencies the effective dielectric constant is essentially constant. At intermediate frequencies its values begin to monotonically increase and eventually approach the values of the dielectric constant of the substrate.

Effective dielectric constant versus frequency for typical substrates.

- The initial values (at low frequencies) of the effective dielectric constant are referred to as the static values, and they are given by

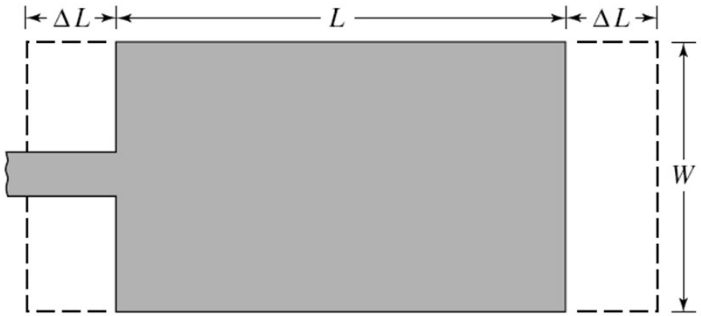
$$W/h > 1$$

$$\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2} \quad [1]$$

-

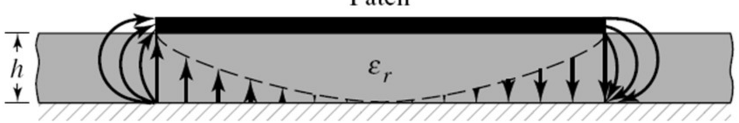
B. Effective Length, Resonant Frequency, and Effective Width

- Because of the fringing effects, electrically the patch of the microstrip antenna looks greater than its physical dimensions.



(a) Top view

For the principal *E*-plane (*xy*-plane), this is demonstrated in the left side Figure, where the dimensions of the patch along its length have been extended on each end by a distance ΔL, which is a function of the effective dielectric constant ε_{reff} and the width-to-height ratio (W/h).



(b) Side view

$$\frac{\Delta L}{h} = 0.412\frac{(\varepsilon_{reff} + 0.3)(\frac{W}{h} + 0.264)}{(\varepsilon_{reff} - 0.258)(\frac{W}{h} + 0.8)} \quad [2]$$

- Since the length of the patch has been extended by Δ L on each side, the effective length of the patch is now ($L = \lambda/2$ for dominant TM_{010} mode with no fringing)

$$L_{eff} = L + 2\Delta L \qquad [3]$$

- For the dominant TM010 mode, the resonant frequency of the microstrip antenna is a function of its length. Usually it is given by

$$(f_r)_{010} = \frac{1}{2L\sqrt{\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = \frac{v_0}{2L\sqrt{\varepsilon_r}}$$

  - where υ_0 is the speed of light in free space.

- If accounting for fringing, it must be modified to include edge effects and should be computed using

$$(f_{rc})_{010} = \frac{1}{2L_{eff}\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} = \frac{1}{2(L + 2\Delta L)\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} = q\frac{1}{2L\sqrt{\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = q\frac{v_0}{2L\sqrt{\varepsilon_r}} \qquad [4]$$

  - where

$$q = \frac{(f_{rc})_{010}}{(f_r)_{010}}$$

- The q factor is referred to as the fringe factor (length reduction factor). As the substrate height increases, fringing also increases and leads to larger separations between the radiating edges and lower resonant frequencies.

C. Design

- Based on the simplified formulation that has been described, a design procedure is outlined which leads to practical designs of rectangular microstrip antennas. The procedure assumes that the specified information includes the dielectric constant of the substrate (ε_r), the resonant frequency (f_r), and the height of the substrate h. The procedure is as follows:
- Specify:

$$\varepsilon_r, f_r \text{ (in Hz), and } h$$

- Determine: W, L
- Design procedure:

1. For an efficient radiator, a practical width that leads to good radiation efficiencies is

$$W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{2v_0}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} \qquad [5]$$

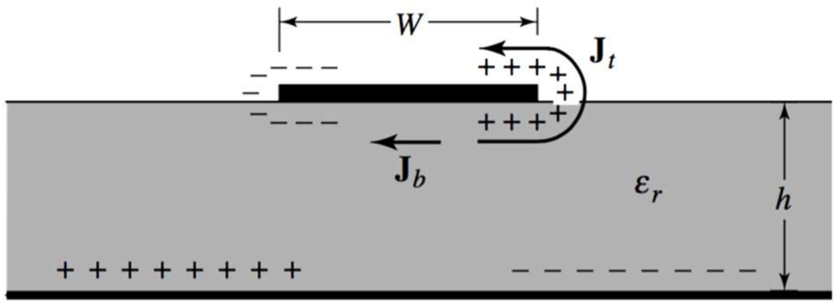
   - where υ_0 is the free-space velocity of light.

2. Determine the effective dielectric constant of the microstrip antenna using [1].
3. Once W is found using [5], determine the extension of the length Δ L using [2].
4. The actual length of the patch can now be determined by solving [4]for L, or

$$L = \frac{1}{2f_r\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} - 2\Delta L$$

4.2 Cavity model

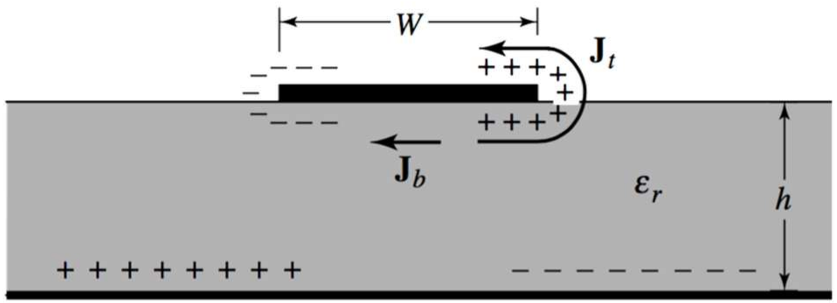
- Microstrip antennas resemble dielectric-loaded cavities, and they exhibit higher order resonances.
- The normalized fields within the dielectric substrate (between the patch and the ground plane) can be found more accurately by treating that region as a cavity bounded by electric conductors (above and below it) and by magnetic walls (to simulate an open circuit) along the perimeter of the patch. This is an approximate model, which in principle leads to a reactive input impedance (of zero or infinite value of resonance), and it does not radiate any power.
- let us attempt to present a physical interpretation into the formation of the fields within the cavity and radiation through its side walls.
- When the microstrip patch is energized, a charge distribution is established on the upper and lower surfaces of the patch, as well as on the surface of the ground plane. The charge distribution is controlled by two mechanisms: an attractive and a repulsive mechanism.



Charge distribution and current density creation on microstrip patch

4.2 Cavity model

- The attractive mechanism is between the corresponding opposite charges on the bottom side of the patch and the ground plane, which tends to maintain the charge concentration on the bottom of the patch.
- The repulsive mechanism is between like charges on the bottom surface of the patch, which tends to push some charges from the bottom of the patch, around its edges, to its top surface.
- The movement of these charges creates corresponding current densities Jb and Jt , at the bottom and top surfaces of the patch, respectively, as shown in Figure
- Since for most practical microstrips the height-to-width ratio is very small, the attractive mechanism dominates and most of the charge concentration and current flow remain underneath the patch. A small amount of current flows around the edges of the patch to its top surface.



Charge distribution and current density creation on microstrip patch

4.2 Cavity model

- However, this current flow decreases as the height-to-width ratio decreases. In the limit, the current flow to the top would be zero, which ideally would not create any tangential magnetic field components to the edges of the patch.
- This would allow the four side walls to be modeled as perfect magnetic conducting surfaces which ideally would not disturb the magnetic field and, in turn, the electric field distributions beneath the patch.
- Since in practice there is a finite height-to-width ratio, although small, the tangential magnetic fields at the edges would not be exactly zero. However, since they will be small, a good approximation to the cavity model is to treat the side walls as perfectly magnetic conducting. This model produces good normalized electric and magnetic field distributions (modes) beneath the patch.
- If the microstrip antenna were treated only as a cavity, it would not be sufficient to find the absolute amplitudes of the electric and magnetic fields. In fact by treating the walls of the cavity, as well as the material within it as lossless, the cavity would not radiate and its input impedance would be purely reactive. Also the function representing the impedance would only have real poles.
- To account for radiation, a loss mechanism has to be introduced, this was taken into account by the radiation resistance Rr and loss resistance RL.

4.2 Cavity model

- These two resistances allow the input impedance to be complex and for its function to have complex poles; the imaginary poles representing, through Rr and RL, the radiation and conduction-dielectric losses.
- To make the microstrip lossy using the cavity model, which would then represent an antenna, the loss is taken into account by introducing an effective loss tangent δeff .The effective loss tangent is chosen appropriately to represent the loss mechanism of the cavity, which now behaves as an antenna and is taken as the reciprocal of the antenna quality factor Q (δeff = 1/Q)
- Because the thickness of the microstrip is usually very small, the waves generated within the dielectric substrate (between the patch and the ground plane) undergo considerable reflections when they arrive at the edge of the patch. Therefore only a small fraction of the incident energy is radiated; thus the antenna is considered to be very inefficient. The fields beneath the patch form standing waves that can be represented by cosinusoidal wave functions.

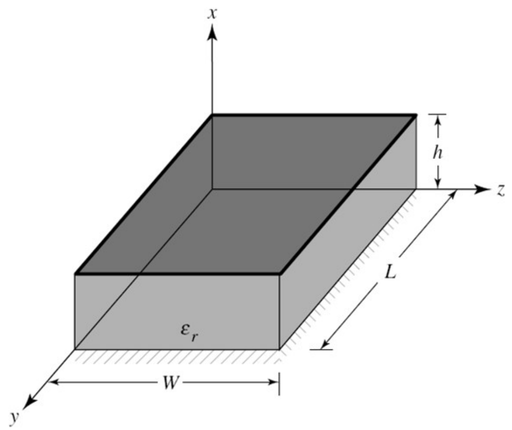
4.2 Cavity model

- Since the height of the substrate is very small ($h \ll \lambda$ where λ is the wavelength within the dielectric), the field variations along the height will be considered constant.
- In addition, because of the very small substrate height, the fringing of the fields along the edges of the patch are also very small whereby the electric field is nearly normal to the surface of the patch. Therefore only TMx field configurations will be considered within the cavity.
- While the top and bottom walls of the cavity are perfectly electric conducting, the four side walls will be modeled as perfectly conducting magnetic walls (tangential magnetic fields vanish along those four walls).

A. Field configurations (modes)-TM^x

- The wave numbers kx, ky, kz are equal to

$$\left.\begin{aligned} k_x &= \left(\frac{m\pi}{h}\right), \quad m = 0, 1, 2, \ldots \\ k_y &= \left(\frac{n\pi}{L}\right), \quad n = 0, 1, 2, \ldots \\ k_z &= \left(\frac{p\pi}{W}\right), \quad p = 0, 1, 2, \ldots \end{aligned}\right\} m = n = p \neq 0$$



  - where m, n, p represent, respectively, the number of half-cycle field variations along the x, y, z directions.

- The resonant frequencies for the cavity are given by

$$(f_r)_{mnp} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{W}\right)^2}$$

- To determine the dominant mode with the lowest resonance, we need to examine the resonant frequencies. The mode with the lowest order resonant frequency is referred to as the dominant mode.
- Placing the resonant frequencies in ascending order determines the order of the modes of operation.

- For all microstrip antennas h<< L and h << W.

① If L > W > h, the mode with the lowest frequency (dominant mode) is the TM^x_{010} whose resonant frequency is given by

$$(f_r)_{010} = \frac{1}{2L\sqrt{\mu\varepsilon}} = \frac{v_0}{2L\sqrt{\varepsilon_r}}$$

- If in addition L > W > L/2 > h, the next higher order (second) mode is the TM^x_{001} whose resonant frequency is given by

$$(f_r)_{001} = \frac{1}{2W\sqrt{\mu\varepsilon}} = \frac{v_0}{2W\sqrt{\varepsilon_r}}$$

- If, however, L > L/2 > W > h, the second order mode is the TM^x_{020}, instead of the TM^x_{001}, whose resonant frequency is given by

$$(f_r)_{020} = \frac{1}{L\sqrt{\mu\varepsilon}} = \frac{v_0}{L\sqrt{\varepsilon_r}}$$

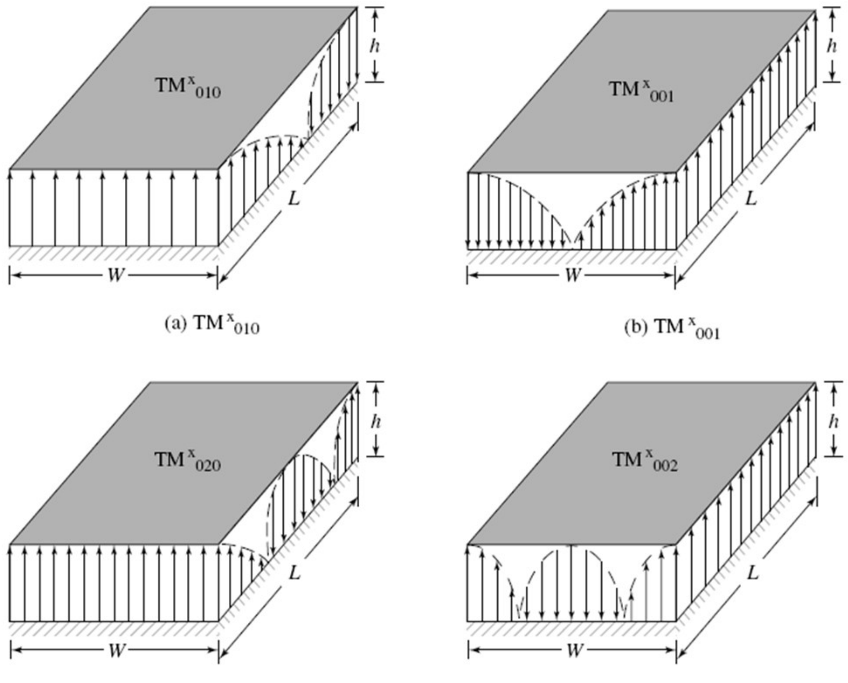
- For all microstrip antennas *h<< L and h << W.*

② If W>*L>h*, the dominant mode is the TM^x_{001} whose resonant frequency is given by

$$(f_r)_{001} = \frac{1}{2W\sqrt{\mu\varepsilon}} = \frac{v_0}{2W\sqrt{\varepsilon_r}}$$

- while if W >W/2 > L > h the second order mode is the TM^x_{002}.

- The distribution of the tangential electric field along the side walls of the cavity for the TM^x_{010}, TM^x_{001}, TM^x_{020} and TM^x_{002} is as shown,



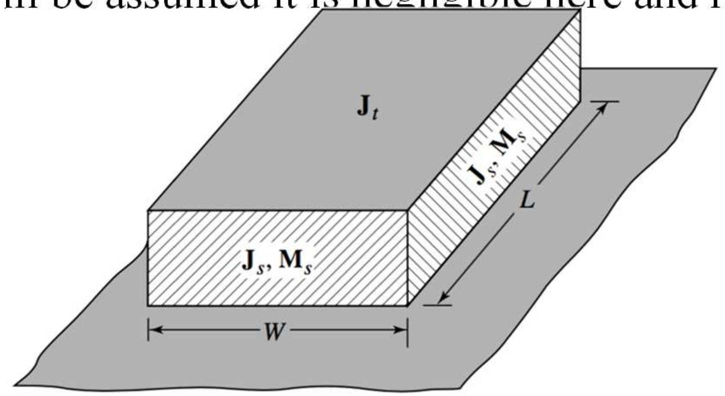
(a) TM^x_{010} (b) TM^x_{001} (c) TM^x_{020} (d) TM^x_{002}

In all of the preceding discussion, it was assumed that there is no fringing of the fields along the edges of the cavity. This is not totally valid, but it is a good assumption. However, fringing effects and their influence were discussed previously,

and they should be taken into account in determining the resonant frequency.

B. Equivalent current densities

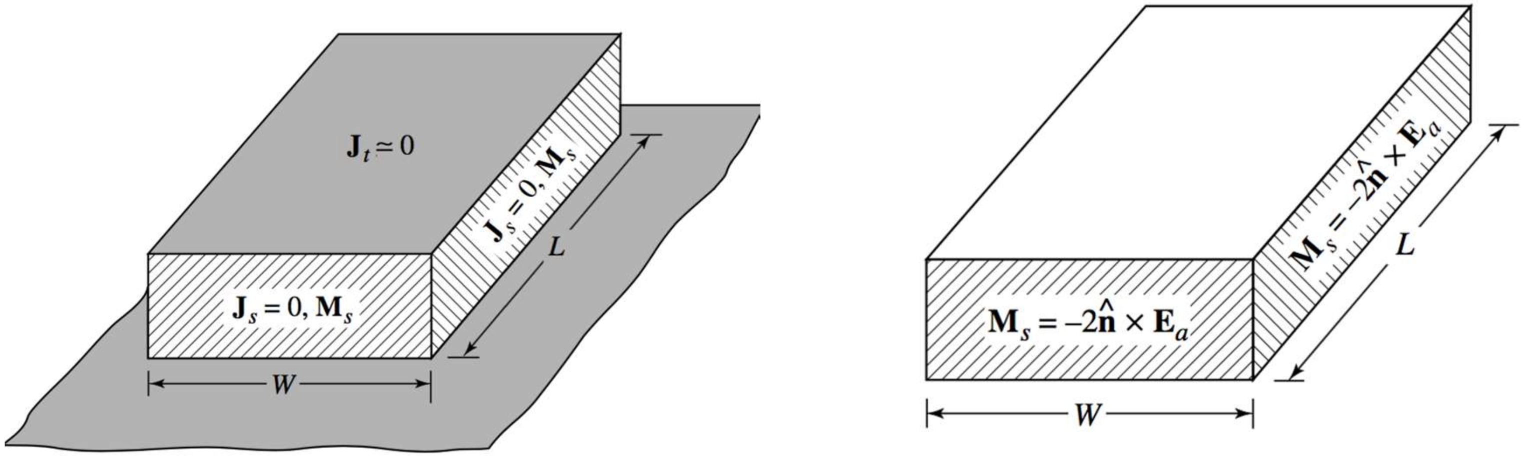
- It has been shown using the cavity model that the microstrip antenna can be modeled reasonably well by a dielectric-loaded cavity with two perfectly conducting electric walls (top and bottom), and four perfectly conducting magnetic walls (sidewalls). It is assumed that the material of the substrate is truncated and does not extend beyond the edges of the patch. The four sidewalls represent four narrow apertures (slots) through which radiation takes place.
- The microstrip patch is represented by an equivalent electric current density J_t at the top surface of the patch to account for the presence of the patch (there is also a current density J_b at the bottom of the patch which is not needed for this model). The four side slots are represented by the equivalent electric current density Js and equivalent magnetic current density M_s, as shown in Figure (a).
- Because it was shown for microstrip antennas with very small height-to-width ratio that the current density J_t at the top of the patch is much smaller than the current density J_b at the bottom of the patch, it will be assumed it is negligible here and it will be set to zero.



(a) $\mathbf{J}_s, \mathbf{M}_s$ with ground plane

B. Equivalent current densities

- Also it was argued that the tangential magnetic fields along the edges of the patch are very small, ideally zero. Therefore the corresponding equivalent electric current density Js will be very small (ideally zero), and it will be set to zero here.
- Thus the only nonzero current density is the equivalent magnetic current density Ms along the side periphery of the cavity radiating in the presence of the ground plane, as shown in Figure (b).
- The presence of the ground plane can be taken into account by image theory which will double the equivalent magnetic current density. Therefore the final equivalent is a magnetic current density of twice around the side periphery of the patch radiating into free-space, as shown in Figure (c).

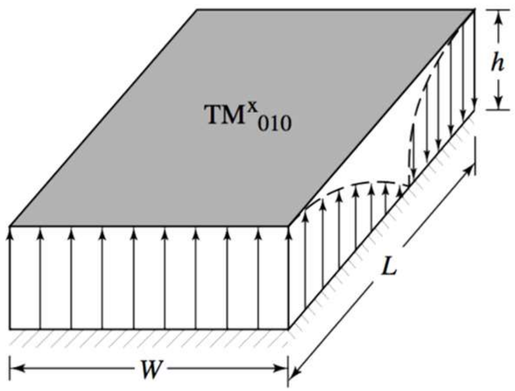


(b) $\mathbf{J}_s = 0$, $\mathbf{M}_s$ with ground plane

(c) $\mathbf{M}_s$ with no ground plane

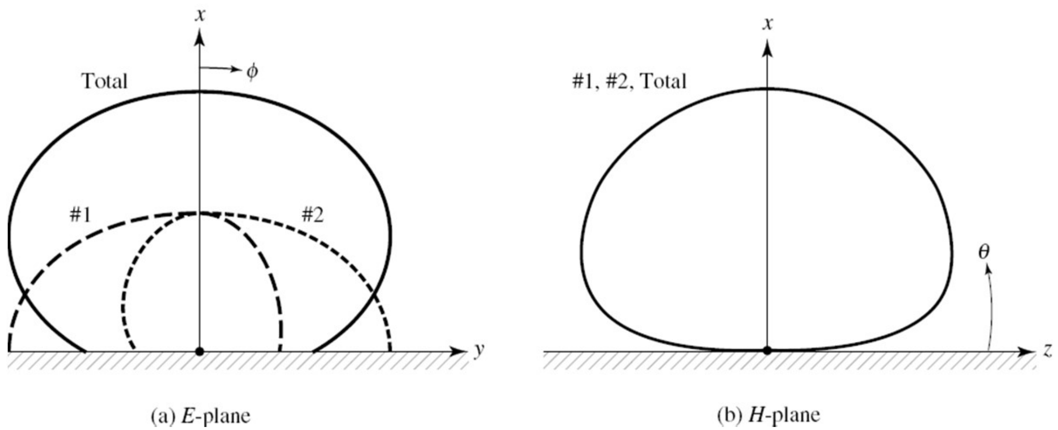
B. Equivalent current densities

- Similarly it will be shown here also that while there are a total of four slots representing the microstrip antenna, only two (the radiating slots) account for most of the radiation; the fields radiated by the other two, which are separated by the width W of the patch, cancel along the principal planes.
- Therefore the same two slots, separated by the length of the patch, are referred to here also as radiating slots.
- The slots are separated by a very low-impedance parallel-plate transmission line of length L, which acts as a transformer. The length of the transmission line is approximately $\lambda/2$, where λ is the guide wavelength in the substrate, in order for the fields at the aperture of the two slots to have opposite polarization.
- The two slots form a two-element array with a spacing of $\lambda/2$ between the elements. It will be shown here that in a direction perpendicular to the ground plane the components of the field add in phase and give a maximum radiation normal to the patch; thus it is a broadside antenna.

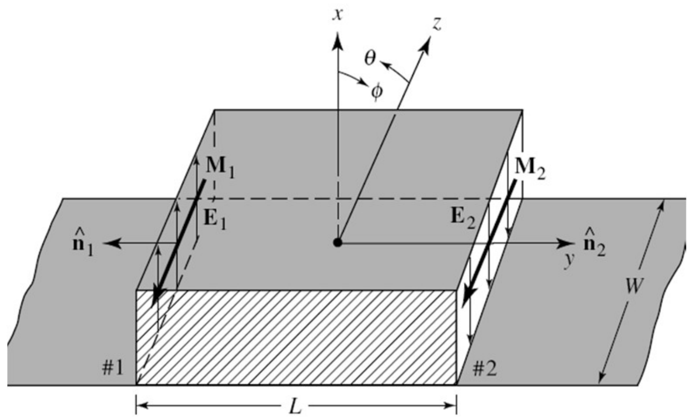


B. Equivalent current densities

- Assuming that the dominant mode within the cavity is the TM^{x}_{010} mode
- each slot radiates the same fields as a magnetic dipole with current density Ms
- the equivalent magnetic current densities along the two slots, each of width W and height h, are both of the same magnitude and of the same phase. Therefore these two slots form a two-element array with the sources (current densities) of the same magnitude and phase, and separated by L. Thus these two sources will add in a direction normal to the patch and ground plane forming a broadside pattern.



Typical E- and H-plane patterns of each microstrip patch slot and of the two together.



Rectangular microstrip patch radiating slots and equivalent magnetic current densities.

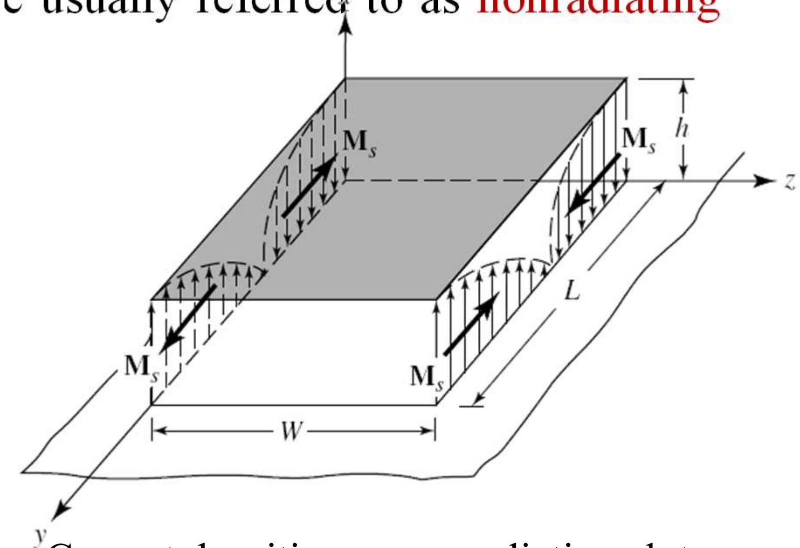
B. Equivalent current densities

- The equivalent current densities for the other two slots, each of length L and height h, are shown in the following Figure. Since the current densities on each wall are of the same magnitude but of opposite direction, the fields radiated by these two slots cancel each other in the principal H-plane.

- Also since corresponding slots on opposite walls are 180◦ out of phase, the corresponding radiations cancel each other in the principal E-plane.

- The radiation from these two side walls in nonprincipal planes is small compared to the other two side walls. Therefore these two slots are usually referred to as nonradiating slots.

- Assuming that the dominant mode within the cavity is the TM^{x}_{010} mode



Current densities on nonradiating slots of rectangular microstrip patch.

4.3 Quality factor, bandwidth, efficiency

- The quality factor, bandwidth, and efficiency are antenna figures-of-merit, which are interrelated, and there is no complete freedom to independently optimize each one. Therefore there is always a trade-off between them in arriving at an optimum antenna performance.
- The quality factor is a figure-of-merit that is representative of the antenna losses. Typically there are radiation, conduction (ohmic), dielectric and surface wave losses. Therefore the total quality factor Q_t is influenced by all of these losses and is, in general, written as

$$\frac{1}{Q_t} = \frac{1}{Q_{rad}} + \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_{sw}}$$

  - Where Q_t = total quality factor; Q_{rad} = quality factor due to radiation(space wave) losses
  - Q_c = quality factor due to conduction (ohmic) losses
  - Q_d = quality factor due to dielectric losses Q_{sw} = quality factor due to surface waves
- For very thin substrates *($h \ll \lambda_0$)* of arbitrary shapes (including rectangular and circular), there are approximate formulas to represent the quality factors of the various losses

$$Q_c = h\sqrt{\pi f \mu \sigma} \qquad Q_d = \frac{1}{tan\delta} \qquad Q_{rad} = \frac{2\omega\varepsilon_r}{hG_t/l}K$$

For a rectangular aperture operating in the dominant TMx010 mode, $K = L/4 \quad G_t/l = \frac{G_{rad}}{W}$

4.3 Quality factor, bandwidth, efficiency

- The Q_{rad} is inversely proportional to the height of the substrate, and for very thin substrates is usually the dominant factor.
- The fractional bandwidth of the antenna is inversely proportional to the Q_t of the antenna, and it is defined

$$\frac{\Delta f}{f_0} = \frac{1}{Q_t}$$

  - However, the above equation may not be as useful because it does not take into account impedance matching at the input terminals of the antenna.
- A more meaningful definition of the fractional bandwidth is over a band of frequencies where the VSWR at the input terminals is equal to or less than a desired maximum value, assuming that the VSWR is unity at the design frequency

$$\frac{\Delta f}{f_0} = \frac{VSWR - 1}{Q_t\sqrt{VSWR}}$$

- In general it is proportional to the volume, which for a rectangular microstrip antenna at a constant resonant frequency can be expressed as

$$BW \sim volume = area \cdot height = length \cdot width \cdot height$$

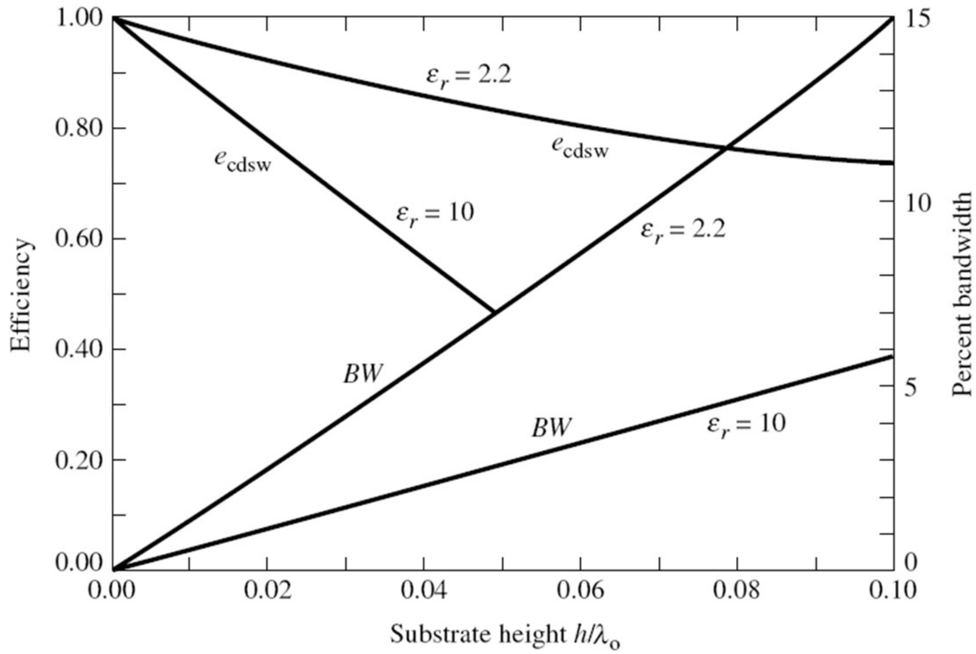
$$\sim \frac{1}{\sqrt{\varepsilon_r}} \frac{1}{\sqrt{\varepsilon_r}} \sqrt{\varepsilon_r} = \frac{1}{\sqrt{\varepsilon_r}}$$

- Therefore the bandwidth is inversely proportional to the square root of the dielectric constant of the substrate.

4.3 Quality factor, bandwidth, efficiency

- The radiation efficiency of an antenna is defined as the power radiated over the input power. It can also be expressed in terms of the quality factors, which for a microstrip antenna can be written as

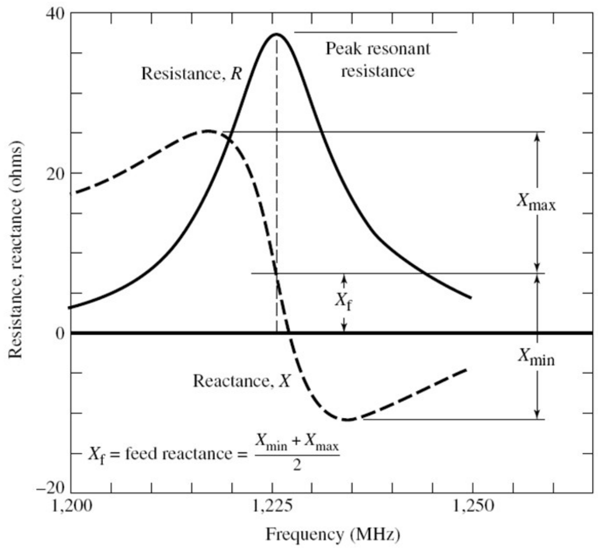
$$e_{cdsw} = \frac{1/Q_{rad}}{1/Q_t} = \frac{Q_t}{Q_{rad}}$$



Efficiency and bandwidth versus substrate height at constant resonant frequency for rectangular microstrip patch for two different substrates.

4.4 Input impedance

- In general, the input impedance is complex and it includes both a resonant and a nonresonant part which is usually reactive. Both the real and imaginary parts of the impedance vary as a function of frequency, and a typical variation is shown:



- A formula that has been suggested to approximate the feed reactance

$$x_f \cong -\frac{\eta k h}{2\pi}[\ln\left(\frac{kd}{4}\right) + 0.577]$$

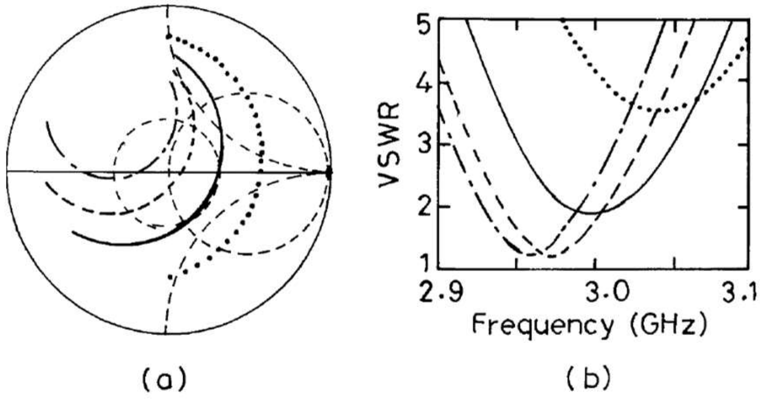
➢ where d is the diameter of the feed probe.

4.5 Parametric study of RMSA

- A coaxial-fed RMSA of *L = 3 cm and W = 4 cm* is considered to study the effects of various parameters on its performance. The probe diameter is taken as 0.12 cm for the 50-V coaxial probe feed using an SMA connector. The substrate parameters are $\varepsilon_r = 2.55$, $h = 0.159$ cm, and $\tan\delta = 0.001$.
- The antenna has been analyzed using commercially available IE3D software based on MoM. The size of the ground plane is considered to be infinite unless finite ground plane size is specified.

4.5.1 Effect of W

- The width *W* of the RMSA has significant effect on the input impedance, BW, and gain of the antenna. For four different values of *W (2, 3, 4, and* 5 cm), the input impedance and VSWR plots for *x*(feed-point locations from the center of the patch) = 0.65 cm are given in the following Figure.



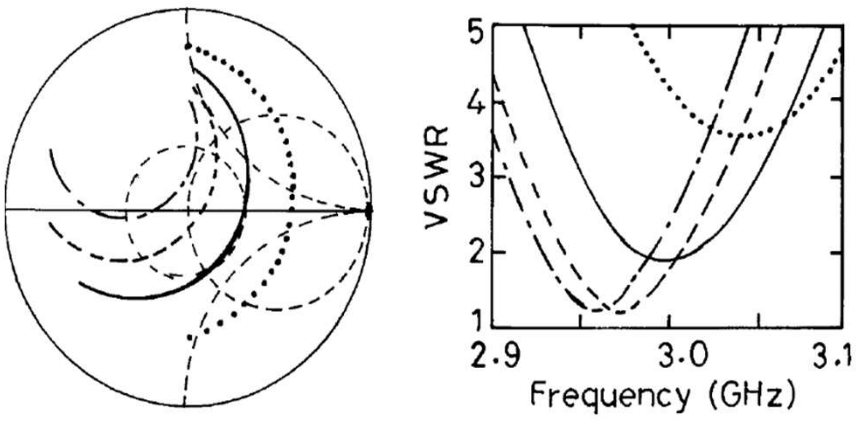
(a) Input impedance and (b) VSWR plots of the RMSA for four different *W:* (…) 2, (——) 3, (- - -) 4, (– - –), 5 cm.

Effect of *W* on the Performance of RMSA
(L = 3 cm, ϵ_r = 2.55, h = 0.159 cm, and tan δ = 0.001)

W (cm)	**x (cm)**	**f_0 (GHz)**	**R_{in} (Ω)**	**BW (MHz)**	**Gain (dB)**	**HPBW (E- and H-planes)**
2	0.35	3.034	57	42	6.2	(105, 86)°
3	0.50	2.993	61	54	6.5	(105, 81)°
4	0.65	2.973	62	64	6.8	(105, 76)°
5	0.75	2.962	53	73	7.0	(105, 70)°

• 4.5.1 Effect of W

- With an increase in *W* from *2 cm* to *5 cm,* the following effects are observed:
  - The resonance frequency decreases from 3.034 GHz to 2.962 GHz due to the increase in ΔL *and* ε_e
  - The input impedance at resonance decreases from 180Ω to 36 Ω, because the radiation from the radiating edge increases, which decreases the radiation resistance.
  - The BW of the antenna increases; however, it is not very evident from these plots, because the feed point is not optimum for the different widths. Accordingly, a better comparison will be obtained when the feed point is optimized for the individual widths.
  - The aperture area of the antenna increases resulting in an increase in the directivity, efficiency, and, hence, gain. The HPBW in the H-plane decreases, whereas it remains almost the same in the E-plane, because the increase in the width is in the H-plane.



(a) (b)

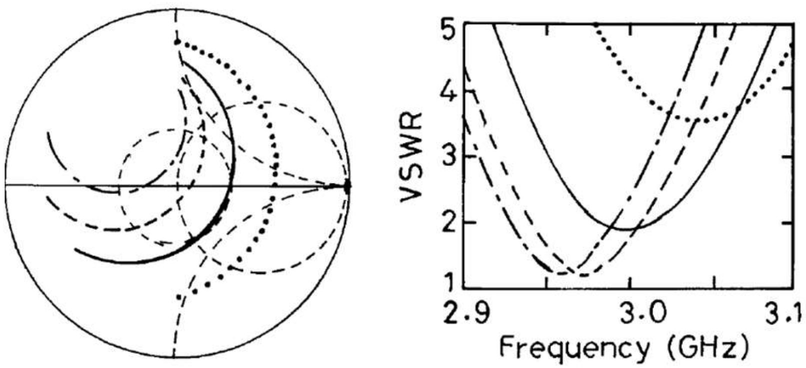
Effect of *W* on the Performance of RMSA
(L = 3 cm, ϵ_r = 2.55, h = 0.159 cm, and tan δ = 0.001)

W (cm)	*x* (cm)	f_0 (GHz)	R_{in} (Ω)	BW (MHz)	Gain (dB)	HPBW (E- and H-planes)
2	0.35	3.034	57	42	6.2	(105, 86)°
3	0.50	2.993	61	54	6.5	(105, 81)°
4	0.65	2.973	62	64	6.8	(105, 76)°
5	0.75	2.962	53	73	7.0	(105, 70)°

4.5.1 Effect of *W*

- With an increase in *W* from *2 cm* to *5 cm,* the following effects are observed:

  - With an increase in *W*, the input impedance decreases, so the feed point is shifted toward the edge to obtain input resistance R_{in} in the range of 50 Ω to 65 Ω;
  - As *W* increases from 2 cm to 5 cm, the value of *x* is increased from 0.35 cm to 0.75 cm, and the BW increases from 42 MHz to 73 MHz.
  - The HPBW in the E-plane remains around 105° , but in the H-plane, it decreases from 86° to 70° .
  - The gain of the RMSA increases from 6.2 dB to 7.0 dB.



(a) (b)

Effect of *W* on the Performance of RMSA
(L = 3 cm, ϵ_r = 2.55, h = 0.159 cm, and tan δ = 0.001)

W (cm)	*x* (cm)	f_0 (GHz)	R_{in} (Ω)	BW (MHz)	Gain (dB)	HPBW (E- and H-planes)
2	0.35	3.034	57	42	6.2	(105, 86)°
3	0.50	2.993	61	54	6.5	(105, 81)°
4	0.65	2.973	62	64	6.8	(105, 76)°
5	0.75	2.962	53	73	7.0	(105, 70)°

4.5.2 Effect of ε_r

- For RMSA with L = 3 cm, W = 4 cm, and h = 0.159 cm, when ε_r is decreased to 1, the resonance frequency increases to 4.541 GHz. The BW of the antenna is 167 MHz for the feed at x = 0.7 cm. This increase in BW is due to a decrease in ε_r and an increase in h/λ_0, because the resonance frequency has increased.
- A better comparison of effect of ε_r is obtained when the antenna is designed to operate in the same frequency range for different values of ε_r .
- Therefore, with change in the dielectric constant from ε_{r1} to ε_{r2}, the L and W dimensions of the RMSA are scaled with a factor of $\sqrt{\varepsilon_{e1}}/\sqrt{\varepsilon_{e2}}$.
- For operation around 3 GHz, the dimensions of the patch for four different values of ε_r (1, 2.55, 4.3, and 9.8) are shown in the following Table.
- The location of the feed point is optimized so that R_{in} is in the range of 50–65Ω for broader BW.
- With a decrease in ε_r from 9.8 to 1, the size of the patch increases and the BW increases from 30 MHz to 74 MHz due to increase in the fringing fields.
- Also, the gain of the antenna increases from 4.4 dB to 10.0 dB due to an increase in the aperture area.

4.5.2 Effect of ε_r

Effect of ϵ_r on the Performance of RMSA
(h = 0.159 cm and tan δ = 0.001)

ϵ_r	L (cm)	W (cm)	x (cm)	f_0 (GHz)	R_{in} (Ω)	BW (MHz)	Gain (dB)
1	4.65	6.2	1.00	2.997	54	74	10.0
2.55	3.0	4.0	0.65	2.974	62	64	6.8
4.3	2.3	3.1	0.40	2.986	52	49	5.6
9.8	1.51	2.0	0.20	3.002	51	30	4.4

- Other effects to be considered: effect of feed point position, effect of h, effect of probe diameter, effect of finite ground plane, effect of loss tangent, effect of cover.

- Conclusions

  1.Feeding methods of MSA.

  2. Analytical methods of MSA.

- Questions
- Design a rectangular microstrip antenna using a substrate FR4 with dielectric constant of 4, thickness of 1mm so as to resonate at 10GHz. (Using the transmission-line model, to calulate the length and width of the patch)

  - How to use the cavity model to analyze a rectangular MSA?
  - The calculation of the rectangular MSA dimensions.

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